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DII-003-016203 Seat No. ________

M. Sc. (Sem. II) (CBCS) Examination

May / June – 2015

Mathematics : Course No. 2003

(Topology - II)

Faculty Code : 003
Subject Code : 016203

Time : 150 Minutes] [Total Marks : **70**

Instructions: (1) There are five questions in this paper.
(2) Each question carries 14 marks.
(3) All questions are compulsory

Q.1 Fill in the blanks: (Each question carries two marks)

(a) A space X is _______ if every open cover of X has finite sub cover.

(b) Every finite subspace of a Hausdorff space is ______________

(c) Every Locally compact Haussdorff space is ___________

(d) The subspace$\mathbb{R} \setminus Q$ of irrationals is not connected because $\mathbb{R} \setminus Q$ is not_________

(e) If X is compact and Y is not compact then $X \times Y$ is ___________

(f) $[0, 1] \times [0, 1]$ with dictionary order topology is connected but not __________-

(g) A space X is connected if and only if X has only __________components.

Q.2 Attempt any two of the following:

(a) Prove that $X \times Y$ is Locally compact if both X and Y are Locally compact spaces. 7

(b) Prove that 7
   (i) Every closed subspace of a Locally compact space is Locally compact.
   (ii) Every compact Hausdorff space is normal.

(c) Prove that Every open subspace of a locally Hausdorff compact space is locally compact. 7

Q.3 All are compulsory

(a) Suppose a filter converges to p. Prove that p is a cluster point of the filter. 6

(b) Suppose every filter on X has a cluster point in X then prove that X is compact. 4

(c) Let X be a space and $x \in X$. Prove that the collection of all neighbourhoods of x is a filter on X. 4

OR

Q.3 All are compulsory

(a) Prove that $X \times Y$ is locally connected if and only if both X and Y are locally connected. 7

(b) Suppose $f : X \rightarrow Y$ is open, continuous and onto. Prove that if X is locally compact then Y is locally compact. 4

(c) Give an example of a limit point compact space which is not compact. 3

Q.4 Attempt any two of the following:

(a) Suppose $f : X \rightarrow Y$ is continuous and onto. Prove that if X is connected then Y is also connected. 7

(b) Define a path connected space. Suppose A and B are path connected subsets of X and $A \cap B \neq \Phi$ then prove that $A \cup B$ is path connected. 7

(c) Suppose C is a component of X and A is a connected subset of X then prove that either A is a subset of C or A is disjoint from C. 7

Q.5 Do as directed: (Each question carries two marks)

(a) Give an open cover of the set $\mathbb{R}$ of real numbers with usual topology which has no finite sub cover.

(b) Give the two subsets of $\mathbb{R}$ (the set of real numbers with standard topology) such that one is closed but not bounded and the other is bounded but not closed.

(c) Let $A = \{1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots\ldots, \frac{1}{n} \ldots\ldots\}$. Is A a compact subset of $\mathbb{R}$? Give reasons for your answer.

(d) Give an example of a Uncountable disconnected space.

(e) Give an example of a compact, Hausdorff space which is denumerable.

(f) Give an example of a finite connected space with four elements.

(g) Give a separation of $\mathbb{R} \setminus \{0\}$.